Grade Level/Course: Algebra 2

Lesson/Unit Plan Name: Writing Exponential Equations with Percents

Rationale/Lesson Abstract: This lesson is in handout form for students to learn about and practice writing exponential equations involving percent increases or decreases. This lesson is given after initial introductory lesson of an exponential function in the form $f(x) = a \cdot b^x$, or similar.

Timeframe: 55 minutes.

Common Core Standard(s):

A-CED.1 Create equations that describe relationships.

F-IF.8b Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$ and classify them as representing exponential growth or decay.*

Instructional Resources/Materials: Copies of handout (pgs. 5-6)
Copies of exit tickets (p. 7)

Activity/Lesson:

Do Now: (5 minutes)

1. Evaluate if a = 4, b = 2, and c = 3:

$$\begin{aligned} & a \cdot b^c \\ &= 4 \cdot 2^3 \\ &= 4 \cdot 8 \\ &= 32 \end{aligned}$$

2. If you have \$100, how much money will you have if you compound this amount by 10% each year for three years straight?

$$\begin{aligned} 100 + 100(.10) &= 110 \\ 110 + 110(.10) &= 121 \\ 121 + 121(.10) &= 133.1 \end{aligned}$$

You will have \$133.10 after 3 years.

Introduction: (5 minutes)

Ask students to read (silently, chorally, volunteer) the title of the lesson and the standard.

Ask students to change each percent to decimal equivalent. Review solutions quickly.

Walk students through the next section. Randomly call on students to read and to answer questions to confirm their understanding.

We Do: (5 minutes)

Help students identify initial amounts and the rate. Cold call students to answer questions like "What is the rate of increase?" and "What is the initial population amount?"

It may be useful to walk students through a few percent problems where they add or subtract the growth/decay from 100%, then change to a decimal. For example, a 5% increase would be 100% + 5% = 105% = 1.05 .

Demonstrate calculator use at this time.

Write out answers in complete sentences to solidify student meaning.

You Try: (5 minutes)

Emphasize the expectation that students should write a sentence explaining their answer.

Teamwork: (30 minutes)

Students work in teams together to find solutions to the remaining problems.

Some good team norms are:
--Same problem, same time.
--Help each other.
--Math discussions only.
--Team questions only.

Solutions found on pgs. 3-4

Assessment--Exit Ticket: (4 minutes)

Distribute exit tickets, remind students this is quiet time, and collect at the door as student are excused.

Assess your success by quickly checking exit tickets and modifying your next class' lesson accordingly.

Solutions---

24% = 0.24	5% = 0.05	70% = 0.70	7% = 0.07	2.5% = 0.025
100% = 1.0	-34% = -0.34	-3.4% = -0.034	-.5% = -0.005	200% = 2.0

We Do: Write an exponential equation that would model a population of 10,000 increasing by 3% per year.

$$f(t) = 10{,}000(1.03)^t$$

At this rate, find the population after 4 years.

$$f(4) = 10{,}000(1.03)^4$$
$$f(4) \approx 11{,}255$$

After 4 years, there will be a population of about 11,255.

Write an exponential equation that would model a population of 500 decreasing by 25% per year.

$$f(t) = 500(0.75)^t$$

At this rate, find the population after 3 years.

$$f(3) = 500(0.75)^3$$
$$f(3) \approx 211$$

After 3 years, there will be a population of 211.

You Try: Write an exponential equation that would model a stock investment of $300 increasing by 200% per year.

$$f(t) = 300(3)^t$$

At this rate, find the investment amount after 5 years.

$$f(5) = 300(3)^5$$
$$f(5) = 72{,}900$$

After 5 years, the stock investment would be worth $72,900.

Write an exponential equation that would model a bank account balance decreasing by 20% per year with an initial deposit of $1500.

$$f(t) = 1500(0.80)^t$$

At this rate, find the account balance after 2 years.

$$f(2) = 1500(0.80)^2$$
$$f(2) = 960$$

After 2 years, the bank account balance would be $960.

Teamwork: 1. A town has a population of 4,000 and is increasing by 15% per year. Write an exponential equation that can be used to find the town's population after 3 years.

$$f(t) = 4000(1.15)^t$$

Calculate the town's population after 3 years at this rate.

$$f(3) = 4000(1.15)^3$$
$$f(3) \approx 6{,}084$$

After 3 years, the town will have 6,084 people.

2. A house's property value is $400,000 and is decreasing in value by 4% per year. Write an exponential equation that would find the property value of the house after 6 years.

$$f(t) = 400{,}000(.96)^t$$

Calculate the house's value after 4 years at this rate.

$$f(6) = 400{,}000(.96)^6$$
$$f(6) \approx 313{,}103$$

After 6 years, the house would be worth about $313,103.

3. A bank's savings account accrues 1.5% interest per year. Write an equation and calculate the account's balance after 7 years with an initial deposit of $500.

$$f(t) = 500(1.015)^t$$
$$f(7) = 500(1.015)^7$$
$$f(7) = 554.92$$

There will be $554.92 in the savings account after 7 years.

4. Tommy's Twitter followers are decreasing at a rate of 22% per month. Currently he has 120 followers. Write an equation that would help find how many months it will take for his follower numbers to drop to 10? (Careful here!)

$$f(t) = 120(0.78)^t$$

The final number of followers after *t* years is 10, so *f(t)=10:*

$$10 = 120(0.78)^t$$

(This will lead into the next lesson on solving equations using the Power Property of Logarithms.)

5. The population of Mexico is 120 million and its growth rate is 1.12%. Write an equation that would help to find how many years it will take to reach 130 million.

$$130 = 120(1.0112)^t$$

6. The population of Russia is 144 million and it's growth rate is -0.51%. Write an equation that would help find when the population of Russia would reach 100 million.

$$100 = 144(0.9949)^t$$

7. The population of Winnemucca, Nevada, can be modeled by $P(t) = 6191(1.04)^t$, where *t* is the number of years since 1990.

What was the population in 1990?

The population was 6,191 in 1990.

By what percent did the population increase by each year?

The percent increase was 4% per year.

Write an equation that would find the number of years when the population would reach 15,000.

$$15{,}000 = 6191(1.04)^t$$

8. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. Find how much ibuprofen is left in the body after 6 hours.

$$f(6) = 400(0.71)^6$$
$$f(6) \approx 51.24$$

There is about 51 milligrams in the body after 6 hours.

Exit Ticket

The foundation of your house has about 1,200 termites. The termites grow at a rate of about 2.4% per day. Write an equation that would help find how long it would take until the number of termites doubles.

$$2{,}400 = 1{,}200(1.024)^t$$

Writing and Solving Exponential Equations with Percents Name:_____

A.CED.1—Create equations that describe relationships.

F-IF.8b Use the properties of exponents to interpret expressions for exponential functions.

Review: Changing percent to decimals:

24% =	5% =	70% =	7% =	2.5% =
100% =	-34% =	-3.4% =	-.5% =	200% =

Recall: $f(t) = a \cdot b^t$ where a is the initial condition and b is the growth/decay rate.

**If you increase something by 3%, then you will have 103% (100 + 3) of the original amount.
**If you decrease something by 12%, then you will have 88% (100 - 12) of the original amount.
**Doubling an amount, increases the amount by 100%, giving you 200% (100 + 100)
**Halving an amount, you will have 50% (100 - 50) of the original amount.

These percentages can be expressed with $f(t) = a \cdot (1 + r)^t$ where *a* is the initial amount, *r* is the rate expressed as a decimal and *t* is the time elapsed. (The 1 represents 100% as a decimal.)

→ If the rate is an increase (growth) then the quantity (1 + r) will be greater than 1 (e.g.: 1 + 0.25 = 1.25)

→ If the rate is a decrease (decay) then the quantity (1 + r) will be less than 1 (eg: 1 + (-0.25) = 0.75).

Let's put all this together:

We Do: Write an exponential equation that would model a population of 10,000 increasing by 3% per year. At this rate, find the population after 4 years. About 11,255 people	Write an exponential equation that would model a population of 500 decreasing by 25% per year. At this rate, find the population after 3 years. About 211 people
You Try: Write an exponential equation that would model a stock investment of $300 increasing by 200% per year. At this rate, find the investment amount after 5 years. $72,900	Write an exponential equation that would model a bank account balance decreasing by 20% per year with an initial deposit of $1500. At this rate, find the account balance after 2 years. $960

Teamwork: 1. A town has a population of 4,000 and is increasing by 15% per year. Write an exponential equation that can be used to find the town's population after 3 years.

Calculate the town's population after 3 years at this rate.

About 6,084 people

2. A house's property value is $400,000 and is decreasing in value by 4% per year. Write an exponential equation that would find the property value of the house after 6 years.

Calculate the house's value after 4 years at this rate.

About $313,103

3. A bank's savings account accrues (increases by) 1.5% interest per year. Write an equation and calculate the account's balance after 7 years with an initial deposit of $500.

$554.92

4. Tommy's Twitter followers are decreasing at a rate of 22% per month. Currently he has 120 followers. Write an equation that would help find how many months it will take for his follower numbers to drop to 10? (Careful here!)

(You'll learn how to solve this soon!) ☺

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What was the population in 1990?

By what percent did the population increase by each year?

Write an equation that would find the number of years when the population would reach 15,000.

8. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. Find how much ibuprofen is left in the body after 6 hours.

About 51 mg

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